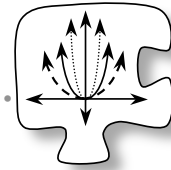


4.1.3 How can I graph it quickly?

Graphing a Parabola Without a Table



You have developed several tools that enable you to transform graphs of parabolas by altering their equations. In the next few lessons, you will use this power to do more with equations and graphs of parabolic functions than ever before. In this lesson, you will figure out how to use your growing knowledge of transforming graphs to make a quick and fairly accurate graph of any parabolic function.

4-34. TRANSFORMING GRAPHS

Use your dynamic graphing tool to support a class discussion about the equation $y = a(x - h)^2 + k$. Refer to the bulleted points below.




- Identify which **parameter** (a , h , or k) affects the orientation, vertical shift, horizontal shift, vertical stretch, and vertical compression of the graph compared to the graph of the parent function $y = x^2$.
- What values stretch the graph vertically? Compress the graph horizontally? Why do those values have these impacts?
- What values cause the graph to flip vertically?
- What values cause the graph to shift to the left? To the right? Why?
- What values cause the graph to shift up or down? Why?
- Are there points on your graph that connect to specific parameters in the equation? Explain.

4-35. For each equation below, predict the coordinates of the vertex, the orientation (opens up or down?), and whether the graph will be a vertical stretch or a compression of $y = x^2$. (Do not use a graphing calculator.) Quickly make a graph based on your predictions. How can you make the shape of your graph accurate without using a table? Be prepared to share your **strategies** with the class.



- | | |
|-------------------------|-------------------------------|
| a. $y = (x + 9)^2$ | b. $y = x^2 + 7$ |
| c. $y = 3x^2$ | d. $y = \frac{1}{3}(x - 1)^2$ |
| e. $y = -(x - 7)^2 + 6$ | f. $y = 2(x + 3)^2 - 8$ |
- g. Now take out your graphing calculator and check your predictions for the equations in parts (a) through (f). Did you make any mistakes? If so, describe the mistake and what you need to do in order to correct it.



METHODS AND MEANINGS

MATH NOTES

Graphing and Standard Form of Parabolas

As you have worked with quadratic functions, equations, and expressions in your previous math courses as well as this course, you have regularly seen two forms of quadratic equations.

One is known as **graphing form** (or **vertex form**), and it looks like $y = a(x - h)^2 + k$. For example, the equation $y = 3(x - 1)^2 - 5$ is in graphing form where $a = 3$, $h = 1$, and $k = -5$.

The other form is known as **standard form** and looks like $y = ax^2 + bx + c$. For example, if $y = 3x^2 - 6x - 2$, then $a = 3$, $b = -6$, and $c = -2$.

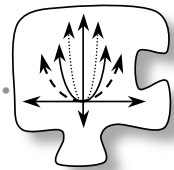


- 4-39. Solve each of the following equations *without using the Quadratic Formula*.
- | | |
|-------------------------|-----------------------------------|
| a. $y^2 - 6y = 0$ | b. $n^2 + 5n + 7 = 7$ |
| c. $2t^2 - 14t + 3 = 3$ | d. $\frac{1}{3}x^2 + 3x - 4 = -4$ |
- e. Zero is one of the solutions of each of the above equations. What do all of the above equations have in common that causes them to have zero as a solution?
- 4-40. Find the vertex of each of the following parabolas by averaging the x -intercepts. Then write each equation in graphing form.
- | | |
|--------------------------|-------------------------|
| a. $y = (x - 3)(x - 11)$ | b. $y = (x + 2)(x - 6)$ |
| c. $y = x^2 - 14x + 40$ | d. $y = (x - 2)^2 - 1$ |

- 4-41. Did you need to average the x -intercepts to find the vertex in part (d) of the preceding problem?
- What are the coordinates of the vertex for part (d)?
 - How do these coordinates relate to the equation?
- 4-42. Scientists can estimate the increase in carbon dioxide in the atmosphere by measuring increases in carbon emissions. In 1998 the annual carbon emission was about eight gigatons (a gigaton is a billion metric tons). Over the last several years, annual carbon emission has been increasing by about one percent.
- At this rate, how much carbon will be emitted in 2010?
 - Write a function, $C(x)$, to represent the amount of carbon emitted in any year starting with the year 2000.
- 4-43. Make predictions about how many places the graph of each equation below will touch the x -axis. You may first want to rewrite some of the equations in a more useful form.
- $y = (x - 2)(x - 3)$
 - $y = (x + 1)^2$
 - $y = x^2 + 6x + 9$
 - $y = x^2 + 7x + 10$
 - $y = x^2 + 6x + 8$
 - $y = -x^2 - 4x - 4$
 - Check your predictions with your calculator.
 - Write a clear explanation describing how you can tell whether the equation of a parabola will touch the x -axis at only one point.
- 4-44. Simplify each of the following expressions. Be sure that your answer has no negative or fractional exponents.
- $64^{1/3}$
 - $(4x^2y^5)^{-2}$
 - $(2x^2 \cdot y^{-3})(3x^{-1}y^5)$
- 4-45. Suppose you have a 3 by 3 by 3 cube. It is painted on all six faces and then cut apart into 27 pieces, each a 1 by 1 by 1 cube. If one of the cubes is chosen at random, what is the probability that:
- Three sides are painted?
 - Two sides are painted?
 - One side is painted?
 - No sides are painted?

4.1.4 How can I model the data?

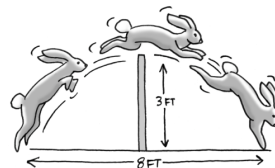
Mathematical Modeling with Parabolas



In the past few lessons, you have determined how to move graphs of parabolas around on a set of axes. In this lesson you will put these new skills to work as you use parabolas and their equations to model situations.

4-46. JUMPING JACKRABBITS

The diagram at right shows a jackrabbit jumping over a three-foot-high fence. To clear the fence, the rabbit must start its jump at a point four feet from the fence.



Sketch the situation and write an equation that models the path of the jackrabbit. Show or explain how you know your sketch and equation fit the situation.

Discussion Points

How can we make a graph fit this situation?

What information do we need in order to find an equation?

How can we be sure that our equation fits the situation?

Further Guidance

- 4-47. What is the shape of the path of the jackrabbit? What kind of equation would best model this situation?
- Sketch the path of the jackrabbit on your own paper. Choose where to place the x - and y -axes in your diagram. Label as many points as you can on your sketch.
 - What point on your graph can tell you about the values of h and k in the equation? Write the values for h and k into the general equation. Is your equation finished?
 - With your team, find a **strategy** to find the exact value of a . Will any of the points on your diagram help? Be prepared to share your **strategy** with the class.
 - What are the domain and range for your model?
 - Did any team in your class get a different equation? If so, write down their equation and show how it can also model the path of the jackrabbit. What choices did that team make differently that resulted in the different equation?

=====
Further Guidance
section ends here.
=====

4-48. When Ms. Bibbi kicked a soccer ball, it traveled a horizontal distance of 150 feet and reached a height of 100 feet at its highest point. Sketch the path of the soccer ball and find the equation of the parabola that models it.

4-49. At the skateboard park, the hot new attraction is the *U-Dip*, a cement structure embedded into the ground. The cross-sectional view of the *U-Dip* is a parabola that dips 15 feet below the ground. The width at ground level, its widest part, is 40 feet across. Sketch the cross sectional view of the *U-Dip*, and find the equation of the parabola that models it.

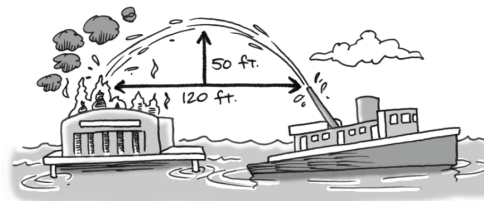
4-50. LEARNING LOG

With your team, discuss all the different forms you know for the equation of a parabola. In your Learning Log, write down each form, along with a brief explanation of how that form is useful. Title this entry, "Forms of a Quadratic Function" and label it with today's date.



4-51. FIRE! CALL 9-1-1!

A fireboat in the harbor is helping put out a fire in a warehouse along the pier. The distance from the barrel of the water cannon to the roof of the warehouse is 120 feet, and the water shoots up 50 feet above the barrel of the water cannon.



Sketch a graph and find the equation of the parabola that models the path of the water from the fireboat to the fire. Give the domain and range for which the function makes sense in relation to the fireboat.

4-52. Draw accurate graphs of $y = 2x + 5$, $y = 2x^2 + 5$, and $y = \frac{1}{2}x^2 + 5$ on the same set of axes. Label the intercepts.

- In the equation $y = 2x + 5$, what does the 2 tell you about the graph?
- Is the 2 in $y = 2x^2 + 5$ also the slope? Explain.

- 4-53. Do the sides of a parabola ever curve back in like the figure at right? Explain your reasoning.



- 4-54. Do the sides of the parabola approach straight vertical lines as shown in the figure at right? (In other words, do parabolas have asymptotes?) Give a reason for your answer.



- 4-55. Find the x - and y -intercepts of the graphs of the two equations below.

a. $y = 2x^2 + 3x - 5$

b. $y = \sqrt{2x - 4}$

- 4-56. The vertex of a parabola (point (h, k)) locates its position on the axes. The vertex serves as a **Locator Point** for a parabola. The other shapes you will be **investigating** in this course also have locator points. These points have different names but the same purpose for each different type of graph.

Sketch graphs for each of the following equations. On each sketch, label the locator point.

a. $y = 3x^2 + 5$

b. $f(x) = -(x - 3)^2 - 7$

- 4-57. If $g(x) = x^2 - 5$, find:

a. $g\left(\frac{1}{2}\right)$

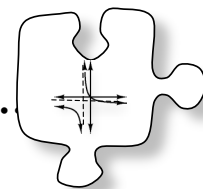
b. $g(h + 1)$

- 4-58. If $g(x) = x^2 - 5$, find the value(s) of x so that:

a. $g(x) = 20$

b. $g(x) = 6$

4.2.1 How can I transform any graph?



Transforming Other Parent Graphs

You have been learning how to move a parabola around a set of axes, write equations, sketch graphs, and model situations. The graph of $y = x^2$ is called the **parent graph** for the family of parabolas because every other parabola can be seen as a transformation of that one graph.

- 4-59. In this **investigation** you will use what you have learned about transforming the graph of $y = x^2$ to transform four new parent graphs. In fact, your team will figure out how to use what you have learned to transform the graph of *any* function!



Your task: As a team, determine how you can make the graph of any function move left, right, up, and down and how you can stretch it vertically, compress it vertically, and flip it. Each team member should choose one of the following parent functions to **investigate**: $y = x^3$, $y = \frac{1}{x}$, $y = \sqrt{x}$, and $y = b^x$. Remember that to **investigate** completely, you should sketch graphs, identify the domain and range, and label any important points or asymptotes. Then graph and write an equation to demonstrate each transformation you find. Finally, you will find a general equation for your family of graphs. (If you are **investigating** $y = b^x$, your teacher will give you a value to use for b .)

Discussion Points

How can we move a parabola?

How can we use our ideas about moving parabolas to move other functions?

What changes can we make to the equation?

Further Guidance

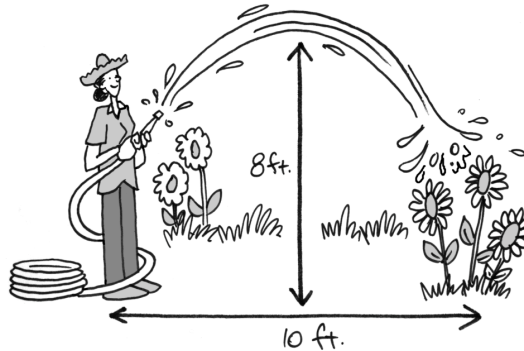
- 4-60. **Investigate** your parent graph.
- Graph your equation on a full sheet of graph paper.
 - As a team, place your parent graphs into the middle of your workspace. For each graph, identify the domain and range and label any important points or asymptotes.
- 4-61. For your parent graph:
- Find and graph an equation that will shift your parent graph left or right.
 - Find and graph an equation that will shift your parent graph up or down.
 - Find and graph an equation that will stretch or compress your parent graph vertically.
 - Find and graph an equation that will flip your parent graph upside-down.
- 4-62. One way of writing a general equation for a parabola is $y = a(x - h)^2 + k$. This equation tells you how to start with the parent graph $y = x^2$ and shift or stretch it to get any other parabola.
- Explain what each parameter (a , h , and k) represents in the graph of a parabola.
 - As a team, write general equations for each given parent equation. Be ready to explain how your general equations work; that is, tell what effect each part has on the orientation (right-side-up or upside-down), relative size (stretched or compressed), horizontal location (left or right shift), and vertical location (up or down shift).

===== *Further Guidance* =====
section ends here.

- 4-63. As a team, organize your work into a large poster that shows clearly:
- Each parent graph you worked with,
 - Examples of each transformation you found, and
 - Each general equation.
- Use tools such as colors, arrows, and shading to show all of the connections you can find. Then add the following problems for other teams to solve:
- Show the graph of a function in your family for which other teams need to find the equation.
 - Give an equation of a function in your family that other teams will graph.

Review & Preview

- 4-64. While watering her outdoor plants, Maura noticed that the water coming out of her garden hose followed a parabolic path. Thinking that she might be able to model the path of the water with an equation, she quickly took some measurements. The highest point the water reached was 8 feet, and it landed on the plants 10 feet from where she was standing. Both the nozzle of the hose and the top of the flowers were 4 feet above the ground. Help Maura write an equation that describes the path of the water from the hose to the top of her plants. What domain and range make sense for the model?



- 4-65. Draw the graph of $y = 2x^2 + 3x + 1$.
- Find the x - and y -intercepts.
 - Where is the line of symmetry of this parabola? Write its equation.
 - Find the coordinates of the vertex.
- 4-66. Change the equation in the previous problem so that the parabola has only one x -intercept.
- 4-67. Simplify each expression. Remember you can simplify radicals by removing perfect square factors (e.g. $\sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$).
- $\sqrt{24}$
 - $\sqrt{18}$
 - $\sqrt{3} + \sqrt{3}$
 - $\sqrt{27} + \sqrt{12}$

4-68. Rewrite each of the following expressions so that your answer has no negative or fractional exponents.

a. $16^{5/4}$ b. $(x^5y^4)^{1/2}$ c. $(x^2y^{-1})(x^{-3}y)^0$

4-69. Harvey's Expresso Express, a drive-through coffee stop, is famous for its great house coffee, a blend of Colombian and Mocha Java beans. Their archrival, Jojo's Java, sent a spy to steal their ratio for blending beans. The spy returned with a torn part of an old receipt that showed only the total number of pounds and the total cost, 18 pounds for \$92.07. At first Jojo was angry, but then he realized that he knew the price per pound of each kind of coffee (\$4.89 for Colombian and \$5.43 for Mocha Java). Show how he could use equations to figure out how many pounds of each type of beans Harvey's used.



4-70. Lilia wants to have a circular pool put in her backyard. She wants the rest of the yard to be paved with concrete.

- If her yard is a 50 ft. by 30 ft. rectangle, what is the largest radius pool that can fit in her yard?
- If the concrete is to be 8 inches thick, and costs \$2.39 per cubic foot, what is the cost of putting in the concrete? No concrete will be used in the pool. (Reminder: $\text{Volume} = (\text{Base Area}) \cdot \text{Depth}$).

4-71. Write the equation $y = x^2 + 7x - 8$ in graphing form. Use what you learned about finding the vertex in Lesson 4.1.3 to help you.

4-72. Consider a line with a slope of 3 and a y-intercept at (0, 2).

- Sketch the graph of this line.
- Write the equation of the line.
- Find the initial term and the next three terms of the sequence $t(n) = 3n - 1$. Plot the terms on a new set of axes next to your graph from part (a) above.
- Explain the similarities and differences between the graphs and equations in parts (a) through (c). Are both continuous?

- 4-73. The gross national product (GNP) was $1.665 \cdot 10^{12}$ dollars in 1960 and it increased at the rate of 3.17% per year until 1989. Use this information to answer each of the questions below. (The number $1.665 \cdot 10^{12}$ is expressed in scientific notation. Written in standard notation, the number is 1,665,000,000,000.)
- What was the GNP in 1989?
 - Write an equation to represent the GNP t years after 1960, assuming that the rate of growth remained constant.
 - Do you think the rate of growth really remains constant? Explain.

4-74. Write each expression in simpler radical form.

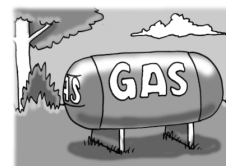
- | | |
|--|-------------------------|
| a. $\sqrt{x} + \sqrt{y} + 5\sqrt{x} + 2\sqrt{y}$ | b. $(2\sqrt{8})^2$ |
| c. $\frac{\sqrt{50}}{\sqrt{2}}$ | d. $\sqrt{\frac{3}{4}}$ |

4-75. Multiply each of the following expressions.

- | | |
|-----------------------|-----------------------|
| a. $2x^2(3x + 4x^2y)$ | b. $(x^3y^2)^4(x^2y)$ |
|-----------------------|-----------------------|

4-76. Sketch a graph and draw the line of symmetry for the equation $y = 2(x - 4)^2 - 3$. What is the equation of the line of symmetry?

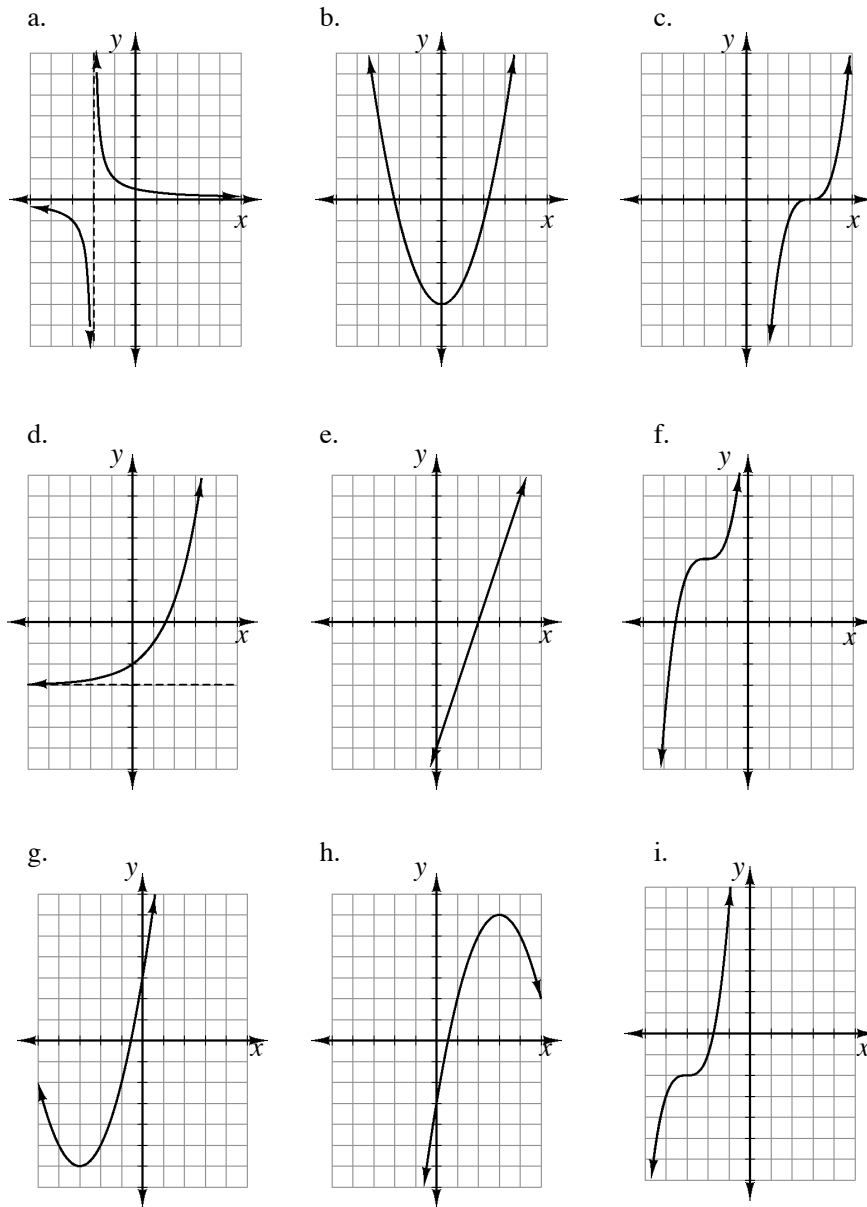
- 4-77. People who live in isolated or rural areas often have their own tanks that hold gas to run appliances like stoves, washers, and water heaters. Some of these tanks are made in the shape of a cylinder with two hemispheres on the ends, as shown in the picture at right. (A hemisphere is half of a sphere, and the volume of a sphere is found by using $V = \frac{4}{3}\pi r^3$.)



The Inland Propane Gas Tank Company wants to make tanks with this shape, but offer different models in different sizes. The cylindrical portion of each of the tanks will be 4 meters long. However, the radius r will vary among the different models.

- One of their tanks has a radius of 1 meter. What is its volume?
- When the radius doubles (to 2 meters), will the volume double? Explain. Then figure out the volume of the larger tank with $r = 2$ m.
- Write an equation that will let Inland Propane Gas Tank Company determine the volume of a tank with any size radius.

4-78. Write a possible equation for each of these graphs. Assume that one mark on each axis is one unit. When you are in class, check your equations on a graphing calculator and compare your results with your teammates.



4-79. By mistake, Jim graphed $y = x^3 - 4x$ instead of $y = x^3 - 4x + 6$. What should he do to his graph to get the correct one?

- 4-80. This is a Checkpoint for solving quadratic equations and for finding the x - and y -intercepts of the graph of a quadratic function.



- Find the x - and y -intercepts for the graph of $y = x^2 + 4x - 17$ without using a graphing calculator.
- Check your answers by referring to the Checkpoint 7 materials located at the back of your book.

If you needed help to solve this problem correctly, then you need more practice in solving quadratic equations and finding the intercepts of a quadratic function. Review the Checkpoint 7 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to find intercepts of quadratic functions easily and accurately.

- 4-81. Simplify each radical expression.

a. $(3\sqrt{2})^2$ b. $\sqrt{\frac{9}{4}}$ c. $\sqrt{\frac{1}{3}}$ d. $(3 + \sqrt{2})^2$

- 4-82. Factor each of the following expressions. Look for the difference of squares and common factors.

a. $4x^2 - 9y^2$ b. $8x^3 - 2x^7$ c. $x^4 - 81y^4$ d. $8x^3 + 2x^7$

e. Did you use a shortcut to factor the expressions in parts (a) through (c)? If so, describe it. If not, what pattern do you see in these expressions? How can you use that pattern to factor quickly?

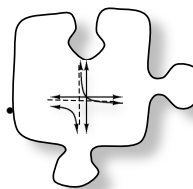
- 4-83. Solve for x : $ax + by^3 = c + 7$.

- 4-84. The slope of \overline{AB} is 5, with points $A(-3, -1)$ and $B(2, n)$. Find the value of n and the distance between points A and B .

- 4-85. Given $f(x) = x^3 + 1$ and $g(x) = (x + 1)^2$:

- Sketch the graphs of the two functions.
- Solve $f(x) = 9$.
- Solve $g(x) = 0$.
- Solve $f(x) = -12$.
- Solve $g(x) = -12$.
- For how many values of x does $f(x)$ equal $g(x)$? Explain.
- Find and simplify an expression for $f(x) - g(x)$.

4.2.2 What is the significance of (h, k) ?



Describing (h, k) for Each Family of Functions

In Lesson 4.2.1, you learned that you can apply your knowledge of transforming parabolas to transform many other parent functions. In this lesson, you will consolidate your knowledge of each of the parent functions that you know and you will identify the importance of the point (h, k) for each parent function and its family.

4-86. Think about the parent graph for parabolas, $y = x^2$.

- Write the equation of a parabola that will be the same as the parent graph, but shifted four units to the right.
- Does the **strategy** you used to move parabolas horizontally also work for other parent graphs? **Justify** your answer.
- You have learned that the general equation for a parabola is $y = a(x - h)^2 + k$. To move the graph of $y = x^2$ h units to the *right*, you replaced x^2 with $(x - h)^2$. Work with your team to **justify why** replacing x with $(x - h)$ moves a graph to the right. Think about multiple representations as you discuss this and be prepared to share your ideas with the class.



4-87. With your team, brainstorm a list of all of the families of functions that you have learned about so far in your study of algebra.

4-88. Obtain copies of the Parent Graph Tool Kit (Lesson 4.2.2 Resource Page) from your teacher. Work with your team to complete a Tool Kit entry for each of the parent graphs you have studied so far in this course. For each parent graph, complete each of the following.

- Name the family of functions.
- Write the equation of the parent function.
- Create a table and graph of the parent function.
- Write the general equation of the family in graphing form.
- Describe the properties of that family of functions.
- State the domain and range of the parent function.
- Describe the significance of the point (h, k) for the family.


- 4-89. What is the equation of the parent graph of a line? Use what you have learned about transforming parent graphs to write the general equation of a transformed line.
- Use this general equation of a line to write the equation of a line with slope $\frac{4}{5}$ that passes through the point $(3, 9)$.
 - A line passes through the points $(-1, 5)$ and $(8, -2)$. Substitute each of these into the general equation to create a system of equations. Now solve this system to find the slope. Is this how you have found slope in the past?

4-90. LEARNING LOG

How can the point (h, k) help you to graph a function from its equation? How can it help you write the equation for a function given its graph? Discuss these questions with your team and then answer them in a Learning Log entry. Be sure to include examples to help you illustrate your ideas. Title this entry “How to use (h, k) ” and label it with today’s date.



MATH NOTES



METHODS AND MEANINGS

Point-Slope Equations for Lines

If you think of $y = x$ as a parent equation, then the general equation for the family of lines can be written as

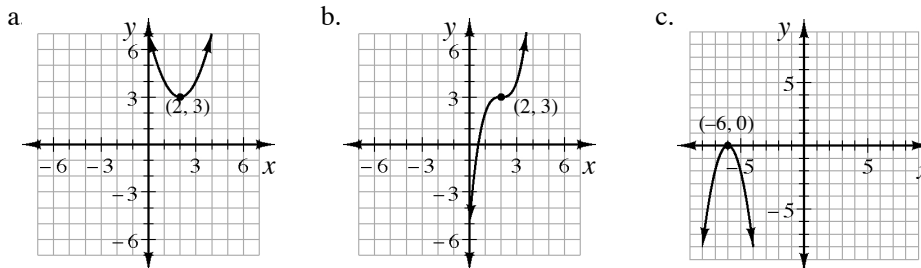
$$y = a(x - h) + k.$$

When this equation is rewritten as $y - k = a(x - h)$ it is often called the **point-slope** form of the equation for a line that contains the point (h, k) and has slope a .

For example, if you know a line contains the point $(7, -8)$ and has slope -4 then the equation can be written $y - (-8) = -4(x - 7)$ or $y + 8 = -4(x - 7)$.



4-91. Use the point (h, k) to help you write a possible equation for each graph shown below.



4-92. Find the domain and range for each of the graphs in the previous problem.

4-93. For each of the following equations, describe how d transforms the parent graph.

a. $y = dx^3$

b. $y = 3x^2 - d$

c. $y = (x - d)^2 + 7$

d. $y = \frac{1}{x} + d$

4-94. Plot each pair of points and find the distance between them. Give answers in both square-root form and as decimal approximations.

a. $(3, -6)$ and $(-2, 5)$

b. $(5, -8)$ and $(-3, 1)$

c. $(0, 5)$ and $(5, 0)$

d. Write the distance you found in part (c) in simplified square-root form.

4-95. Rewrite each of the following expressions so that your answers have no negative or fractional exponents.

a. $5^{-2} \cdot 4^{1/2}$

b. $\frac{3xy^2z^{-2}}{(xy)^{-1}z^2}$

c. $(3m^2)^3(2mn)^{-1}(8n^3)^{2/3}$

d. $(5x^2y^3z)^{1/3}$

4-96. Solve each equation for x (that is, put it in $x = \underline{\hspace{1cm}}$ form).

a. $y = 2(x - 17)^2$

b. $y + 7 = \sqrt[3]{x + 5}$

4-97. Where do the following pairs of lines intersect?

a. $y = 5x - 2$
 $y = 3x + 18$

b. $y = x - 4$
 $2x + 3y = 17$

4-98. Graph these two lines on the same set of axes: $y = 2x$ and $y = -\frac{1}{2}x + 6$.

- Find the x - and y -intercepts for each equation.
- Shade the region bounded by the two lines and the x -axis.
- What are the domain and range of the region? How did you find these values?
- Find the area of this region. Round your answer to the nearest tenth.